

# FORTSCHRITT- BERICHTE **VDI**

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## **High Accuracy Compressibility Factor Calculation for Natural Gases and Similar Mixtures by Use of a Truncated Virial Equation**

Reihe **6**: Energieerzeugung

Nr. **231**

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This monograph provides a detailed account of the concept, development, performance and use of the GERG virial equation. This equation predicts the compressibility factor  $Z(p,T)$  for natural gas-type mixtures (specified by a 13-component molar composition) within the respective pressure and temperature ranges of 0 to 12 MPa (0 to 120 bar) and 265 to 335 K (−8 to 62 °C) with an expectation accuracy of 0.1%. — Coefficients used in the GERG virial equation have been correlated using a databank of over 13,000 high quality compressibility factor experimental measurements. The GERG equation describes a subset of some 4,500 data points for natural gases, which were not used in the correlation development, with a root-mean-square error of 0.055%. Evidence is given that performance is better than that of the GRI-SuperZ (AGA 8) equation within the stated range of applicability.

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- page iv - page number list - "92" should read "94"  
"113" should read "112"  
"114" should read "113"
- page v - first line - "IMPLEMENTATION" is misspelled
- page 17 - ref [25] date - "1966" should read "1965"  
ref [60] date - "1935" should read "1936"
- page 23 - data counts - "73" should read "72"  
"493" should read "492"
- page 36 - ref [60] date - "1935" should read "1936"
- page 57 - penultimate line - first "is" should read "are"
- page 61 - fifth line down - delete second "in"
- page 75 - fourth line down - "80" should read "8"
- page 77 - 18th line down - insert "the" before "GERG"
- page 88 - 15th line down - "(N37)" should read "(N37,  
N79)"
- page 92 - 18th line down - "factor" should read "factors"
- page 97 - figure 5.16 - faulty printing of data point  
for 280 K at 11 MPa
- page 112 - fifth line up - insert "," before "firstly"
- page 148 - reference 33(a) - run together lines 2 and 3
- page 149 - reference 45 - "1251" should read "1250"
- page 158 - first definition  
of N - incorrect line spacing of text

GERG TECHNICAL MONOGRAPH 2 (1988)HIGH ACCURACY COMPRESSIBILITY FACTOR CALCULATION FOR NATURAL  
GASES AND SIMILAR MIXTURES BY USE OF A TRUNCATED  
VIRIAL EQUATION

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on behalf of

Programme Committee No. 1  
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GERG TECHNICAL MONOGRAPH 2 (1988)HIGH ACCURACY COMPRESSIBILITY FACTOR CALCULATION FOR NATURAL GASES AND  
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Abstract

This Monograph provides a detailed account of the concept, development, performance and use of the GERG virial equation. This equation predicts the compressibility factor  $Z(p,T)$  for natural gas-type mixtures (specified by a 13-component molar composition) within the respective pressure and temperature ranges of 0 to 12 MPa (0 to 120 bar) and 265 to 335 K (-8 to 62°C) with an expectation accuracy of 0.1%.

Coefficients used in the GERG virial equation have been correlated using a databank of over 13,000 high quality compressibility factor experimental measurements. The GERG equation describes a subset of some 4,500 data points for natural gases, which were not used in the correlation development, with a root-mean-square error of 0.055%. Evidence is given that performance is better than that of the GRI-SuperZ (AGA8) equation within the stated range of applicability.

Numerical values for all coefficients needed to implement the equation are given, and computer program listings are provided. A simplified version of the GERG virial equation, requiring less comprehensive analytical input data, will be described elsewhere.

The GERG virial equation was developed at the van der Waals laboratory of the University of Amsterdam, under contract to, and with specific guidance from, the Groupe Européen de Recherches Gazières.

Résumé

Cette monographie donne une description détaillée du développement, des possibilités et de l'utilisation de l'équation de viriel du GERG. Cette équation prévoit le facteur de compressibilité  $Z(p,T)$  des gaz naturels (spécifiés par la donnée de la composition molaire à 13 composants) à des pressions comprises entre 0 et 12 MPa (0 à 120 bar) et des températures allant de 265 à 335 K (-8 à 62°C) avec une précision meilleure que 0,1%.

L'équation de viriel du GERG a été validée pour plus de 4400 données du facteur de compressibilité sur des gaz variés (gaz naturels et mélanges multicomposants) extraites d'une banque de données comprenant plus de 13000 mesures de haute qualité.

L'équation a été développée au laboratoire van der Waals de l'université d'Amsterdam sous contrat avec le Groupe Européen de Recherches Gazières (GERG) et sous sa direction.

Une version simplifiée de l'équation, nécessitant des données d'entrée moins complètes sera décrite dans un autre document.

Zusammenfassung

Diese Monographie beschreibt detailliert das Konzept, die Entwicklung, die Genauigkeit und den Gebrauch der GERG-Virialgleichung. Die Gleichung bestimmt Realgasfaktoren  $Z(p,T)$  für Erdgase und Erdgasgemische (gegeben durch eine 13-komponentige molare Gaszusammensetzung) innerhalb eines Druckbereiches von 0 bis 12 MPa (0 bis 120 bar) und eines Temperaturbereiches von 265 bis 335 K (-8 bis 62°C) mit einer Unsicherheit von weniger als 0,1%.

Die GERG-Virialgleichung basiert auf einer Datenbank von über 13000 höchstgenauen gemessenen Realgasfaktoren. Die Gleichung berechnet jene 4500

Datenpunkte für Erdgase, welche bei der Aufstellung der Gleichung nicht benutzt worden sind, mit einem mittleren quadratischen Fehler von ungefähr 0,055%.

Die numerischen Werte für alle Koeffizienten, die für die Implementierung der Gleichung benötigt werden, sind angegeben. Eine vereinfachte Version der GERG-Virialgleichung, die als Eingabe nur eine Teilanalyse benötigt, wird in einem weiteren GERG Technical Monograph beschrieben.

Die GERG-Virialgleichung wurde vom van der Waals-Laboratorium der Universität Amsterdam im Auftrag und unter der Leitung der "Groupe Européen de Recherches Gazières" entwickelt.

### Samenvatting

Deze monografie geeft een gedetailleerde beschrijving van het concept, de ontwikkeling, de toepassing en het gebruik van de viriële GERG-vergelijking. Deze vergelijking voorspelt de samendrukbaarheidsfactor  $Z(p,T)$  voor aardgasmengsels (gespecificeerd door een molaire samenstelling van 13 componenten) binnen de respectievelijke druk- en temperatuurbereiken van 0 tot 12 MPa (0 tot 120 bar) en 265 tot 335 K (-8 tot 62°C) met een te verwachten nauwkeurigheid van 0.1%.

Op basis van een gegevensbank van meer dan 13,000 experimentele hoge kwaliteitsmetingen van de samendrukbaarheidsfactor, beschrijft de GERG-vergelijking deze 4,500 gegevenspunten voor aardgas met een R.M.S.-afwijking van 0.055%.

De numerieke waarden voor alle coëfficiënten, nodig om de vergelijking toe te passen, worden gegeven. Een vereenvoudigde versie van de viriële GERG-vergelijking die minder uitgebreide analytische gegevens vereist, wordt elders beschreven.

De viriële GERG-vergelijking werd door het van der Waals laboratorium (Universiteit Amsterdam) ontwikkeld onder contract en met de specifieke leiding van de "Groupe Européen de Recherches Gazières".

### Sommario

Questa monografia fornisce una descrizione dettagliata circa i criteri di sviluppo, le prestazioni e l'utilizzo dell'equazione viriale GERG. Tale equazione calcola il fattore di compressibilità  $Z(p,T)$  per una miscela di gas naturale (definita attraverso la composizione molare di 13 componenti) in un campo di pressione compreso tra 0 e 120 bar ed una temperatura tra 265 e 335 K (da -8 a 62°C) con una incertezza prevista di 0,1%.

I coefficienti usati nell'equazione viriale GERG sono stati correlati utilizzando una banca dati di altre 13,000 valori sperimentali del fattore di compressibilità misurati con elevata precisione. L'equazione GERG è stata convalidata con più di 4,400 punti per il gas naturale la cui radice quadrata dell'errore medio è di 0,055%. I risultati indicano che tale prestazione è migliore di quella fornita dall'equazione GRI-SuperZ (AGA8) nel campo di applicazione sopra definito.

Una versione semplificata dell'equazione, che necessita di un ridotto numero di dati di input, sarà descritta in un altro documento.

L'equazione è stata sviluppata dal laboratorio van der Waals dell'università di Amsterdam su contratto con il GERG (Groupe Européen de Recherches Gazières) e con la supervisione dello stesso gruppo.

## Foreword

Since the publication of the first GERG Technical Monograph No.TPC/1 "Some Thermophysical Constants of Components of Natural Gas and Cognate Fluids" in 1986, changes have been made in the administrative structure and procedures through which collaborative research is supported within the European natural gas community.

The work described in this Monograph was started several years ago within the GERG Thermodynamics Research Committee (which produced GERG TPC/1) in response to a strongly perceived need for a common means of calculating the compressibility factor of natural gases with higher accuracy than hitherto generally achieved, and applicable to reasonably wide ranges of temperature, pressure and composition. Upon reorganisation of GERG, the Thermodynamics Research Committee ceased to exist; however, a programme to support the completion of this important work had already been agreed by the GERG Plenary Group and, consequently, this single task was assigned to a newly-formed Working Group reporting to the GERG Programme Committee No.1 - Production, Supply and Gas Properties. In large measure continuity was assured by constituting the Working Group to include several members who had previously been members (or attendees) of the former Thermodynamics Research Committee.

The Working Group members hope that the method of calculation presented here in GERG TM 2, which they have developed in collaboration with the van der Waals Laboratory (University of Amsterdam) will enjoy wide acceptance throughout the World's gas utilities, most especially within Europe, as a recognised means of high quality prediction of compressibility factor.

## Acknowledgements

A number of our present and former colleagues have been associated with parts of the work described in this Monograph.

We are especially grateful for the steadfast support and encouragement given by Dr. A. Melvin, initially as Chairman of the GERG Thermodynamics Research Committee and, latterly, as Chairman of the GERG Programme Committee No.1.

Most of the newer experimental data used in the development of the GERG Virial Equation were measured in the Gasunie and Ruhrgas research laboratories; we thank in particular S.R. Reintsema and H.M. Hinze for their diligence in pursuit of appropriate high quality data.

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